MODULE - 3

PROPERTIES OF PLANAR SURFACE

Centroid of a plane is the point where the area can be assumed to be concentrated. Centroid is the geometrical center of the plane. Centre of gravity is the point where mass of a body can be assumed to be concentrated.

Centroid of Some Simple Shapes

1) Rectangle

\[
\text{Centroid } \left( \frac{b}{2}, \frac{h}{2} \right)
\]

2) Square

\[
\text{Centroid } \left( \frac{a}{2}, \frac{a}{2} \right)
\]
3) Triangle

![Diagram of a triangle with centroid at \((b/2, h/3)\)]

4) Circle

![Diagram of a circle with centroid at \((0, 0)\)]

5) Semicircle

![Diagram of a semicircle with centroid at \((0, 4r/3\pi)\)]

For a semicircle, the centroid is \(4r/3\pi\) from the base.

6) Sector

![Diagram of a sector with centroid at \((4r/3\pi, 4r/3\pi)\)]
Centroid of Composite Shape

for a composite shape centroid is given by

\[
(x, y) = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}
\]

\[
y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}
\]

Determine the centroid of given I-section

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Sl.No} & A_i \text{ (mm}^2\text{)} & x_i \text{ (mm)} & y_i \text{ (mm)} & A_i x_i & A_i y_i \\
\hline
1 & 12 \times 4 = 48 & \frac{12}{2} = 6 & \frac{4}{2} = 2 & 288 & 96 \\
2 & 10 \times 2 = 20 & \frac{12}{2} = 6 & \frac{10}{2} + 4 = 9 & 120 & 180 \\
3 & 12 \times 4 = 48 & \frac{12}{2} = 6 & 10 + 4 + 6 = 18 & 288 & 768 \\
\hline
\text{Total} & 116 & \text{ } & \text{ } & 699 & 1044 \\
\hline
\end{array}
\]

\[
x = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3 + \cdots}
= \frac{96 + 120 + 288 + 288}{48 + 20 + 48}
= \frac{696}{116} = 6 \text{ mm}
\]
\[
\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{96 + 180 + 488}{116} = 9 \text{ mm}
\]

Centroid of the given I-section is \((6,9)\)

\[
? \quad \text{Determine centroid of given T-shape}
\]

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{480}{80} = 6 \text{ mm}
\]

\[
\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{850}{80} = 10.625 \text{ mm}
\]

Centroid of T-section \((6,10.6)\)
Determine the centroid of the given composite section.

![Composite section diagram](image)

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>( A_i ) (mm(^2))</th>
<th>( x_i ) (mm)</th>
<th>( y_i ) (mm)</th>
<th>( A_i x_i )</th>
<th>( A_i y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>15x4=60</td>
<td>15/2=7.5</td>
<td>4/2=2</td>
<td>-450</td>
<td>120</td>
</tr>
<tr>
<td>2.</td>
<td>8x2=16</td>
<td>2/2=1</td>
<td>8+4=8</td>
<td>16</td>
<td>128</td>
</tr>
<tr>
<td>3.</td>
<td>10x2=20</td>
<td>10/2=5</td>
<td>8+4+2=13</td>
<td>100</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>96</td>
<td></td>
<td></td>
<td>566</td>
<td>508</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{566}{96} = 5.9 \text{ mm}
\]

\[
\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{508}{96} = 5.3 \text{ mm}
\]

The centroid of the composite section is \((5.9, 5.3)\).
Determine the centroid of area shown in Figure.

\[ \Delta \text{involved area more at } y \]

## A.

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>( A_i ) (cm²)</th>
<th>( x_i ) (cm)</th>
<th>( y_i ) (cm)</th>
<th>( A_i x_i )</th>
<th>( A_i y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{\pi \cdot 2^2}{2} ) = 6.3</td>
<td>2 - ( \frac{4 \cdot 2}{3 \pi} ) = 8.15</td>
<td>2</td>
<td>7.2</td>
<td>12.6</td>
</tr>
<tr>
<td>2</td>
<td>6 \times 4 = 24</td>
<td>2 + 3 = 5</td>
<td>2</td>
<td>120</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{2} \times 4 \times 3 = 6 )</td>
<td>2 + ( \frac{6 + 3}{2} ) = 9.15 ( q )</td>
<td>( \frac{4 \times 1.33}{2} ) = 5.29</td>
<td>( \frac{7.98}{3} ) = 2.66</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>36.3</strong></td>
<td></td>
<td></td>
<td><strong>181.2</strong></td>
<td><strong>68.58</strong></td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{181.2}{36.3} = 4.9 \text{ cm}
\]

\[
\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{68.58}{36.3} = 1.9 \text{ cm}
\]
Determine the centre of gravity of the plane shown in figure.

![Diagram of a plane with dimensions](image)

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Ai (cm²)</th>
<th>xi (cm)</th>
<th>yi (cm)</th>
<th>Aixi</th>
<th>Aiyi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{2} \times 10 \times 15 = 75$</td>
<td>10 - $\frac{10}{3}$ = 6.67</td>
<td>15 - $\frac{15}{3}$ = 10</td>
<td>500.25</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>$20 \times 15\frac{3}{2} = 300$</td>
<td>$10 + \frac{20}{2} = 20$</td>
<td>15 - $\frac{15}{2} = 7.5$</td>
<td>6000.0</td>
<td>2250</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi \times 5^2}{2} = 39.3$</td>
<td>$10 + 5 + 5 = 20$</td>
<td>8 x $\frac{4 \times 3}{3}! = 4 \times 3 = 12$</td>
<td>-786</td>
<td>-83.85</td>
</tr>
<tr>
<td>Total</td>
<td>$1849 \times 8 = 335.7$</td>
<td></td>
<td></td>
<td>4686.2</td>
<td>2916.7</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{5714.2}{335.7} = 17.02 \text{ cm}
\]

\[
\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{2916.7}{335.7} = 8.7 \text{ cm}
\]
Locate the centroid of the area shown in the figure.

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>$A_i$ (m²)</th>
<th>$x_i$ (m)</th>
<th>$y_i$ (m)</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80 x 40</td>
<td>40</td>
<td>20</td>
<td>1680000</td>
<td>640000</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{4} \times \frac{20^2}{4}$</td>
<td>$\frac{4 \times 20}{3\pi}$</td>
<td>$\frac{40 - 4 \times \frac{20}{3\pi}}{3\pi}$</td>
<td>2669.8</td>
<td>9894.15</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{2} \times 40 \times 40$</td>
<td>$\frac{40 + 40}{2}$</td>
<td>$\frac{40}{3}$</td>
<td>53360</td>
<td>10640</td>
</tr>
<tr>
<td>Total</td>
<td>2085.9</td>
<td>66.7</td>
<td>-</td>
<td>71996.2</td>
<td>43436</td>
</tr>
</tbody>
</table>

$\bar{x} = 34.8 \text{ m}$

$\bar{y} = 20.5 \text{ m}$
Determine the centroid of the given shaded area.

\[ y \]
\[ x \]

\[ \text{A} \]

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>( A_i ) (cm²)</th>
<th>( x_i ) (cm)</th>
<th>( y_i ) (cm)</th>
<th>( A_i x_i )</th>
<th>( A_i y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{\pi r^2}{4} ) [= \frac{\pi \times 10^2}{4} ] (= 78.5 )</td>
<td>( \frac{4 \times y}{3 \pi} ) (= 4.2 )</td>
<td>( \frac{4 \times y}{3 \pi} ) (= 4.2 )</td>
<td>(-329.7)</td>
<td>(-329.7)</td>
</tr>
<tr>
<td>2</td>
<td>( 10 \times 10 ) (= 100 )</td>
<td>( 5 )</td>
<td>( 5 )</td>
<td>( 500 )</td>
<td>( 500 )</td>
</tr>
<tr>
<td>Total</td>
<td>21.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{170.3}{210.5} = 0.808 \; \text{cm} \]

\[ \bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{170.3}{210.5} = 0.808 \; \text{cm} \]
Determine the centroid.

---

![Diagram](image)

---

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>$A_i$ (m$^2$)</th>
<th>$x_i$ (m)</th>
<th>$y_i$ (m)</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$120 \times 70 = 8400$</td>
<td>60</td>
<td>35</td>
<td>504000</td>
<td>294000</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1440 \times 20^2}{2} = 1413.7$</td>
<td>$90 + \frac{4 \times 60}{2} = 114.0$</td>
<td>$90 - \frac{4 \times 30}{3\pi} = 57.3$</td>
<td>$84822.4$</td>
<td>$81000.801$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{2} \times 20 \times 40 = 400$</td>
<td>$80 + \frac{20}{2} = 90$</td>
<td>$\frac{40}{3} = 13.3$</td>
<td>$36000$</td>
<td>$5320$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6586.3</strong></td>
<td></td>
<td></td>
<td><strong>383178</strong></td>
<td><strong>207674.9</strong></td>
</tr>
</tbody>
</table>

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{383178}{6586.3} = 58.17 \text{ m}$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{207674.9}{6586.3} = 31.53$$
Three forces 500N, 400N, 800N are acting along three diagonals of adjacent faces of a cube of side 2m in Fig. Determine resultant of force.

A. Coordinates of A \((2,0,2)\)
   "      \((2,2,0)\)
   "      \((0,2,2)\)

Unit vector of \(OA\) = \(\frac{2i+0j+2k}{\sqrt{2^2+2^2}}\) = 0.71i + 0.71k

Force vector along \(OA\) = 500 \((0.71i + 0.71k)\) = 355i + 355k

Unit vector along \(OB\) = \(\frac{2i+2j}{\sqrt{2^2+2^2}}\) = 0.71i + 0.71j

Force vector along \(OB\) = 400 \((0.71i + 0.71j)\) = 449i + 449j

Unit vector of \(OC\) = \(\frac{2j+2k}{\sqrt{2^2+2^2}}\) = 0.71j + 0.71k

Force vector along \(OC\) = 800 \((0.71j + 0.71k)\) = 569j + 569k
Resultant force \( \mathbf{R} = \mathbf{F}_{OA} + \mathbf{F}_{OB} + \mathbf{F}_{OC} \)
\[ = (355i + 355k) + (497i + 497j) + (569i + 569j) \]
\[ = 852i + 1066j + 924k \]

Magnitude of resultant = \( \sqrt{852^2 + 1066^2 + 924^2} \)
\[ = 1648 \text{ N} \]

Direction cosines:
\[ \cos \theta_x = \frac{\mathbf{R}_x}{\mathbf{R}} = \frac{852}{1648} \]
\[ \theta_x = 58.87^\circ \]

\[ \cos \theta_y = \frac{\mathbf{R}_y}{\mathbf{R}} = \frac{1066}{1648} \]
\[ \theta_y = 49.7^\circ \]

\[ \cos \theta_z = \frac{\mathbf{R}_z}{\mathbf{R}} = \frac{924}{1648} \]
\[ \theta_z = 55.9^\circ \]
A rectangular concrete slab supports loads at the four corners as shown in figure. Determine the resultant of these forces and the point of application of the resultant.
\[
\vec{F}_0 = -125\text{j} \\
\vec{F}_A = -250\text{j} \\
\vec{F}_B = -150\text{j} \\
\vec{F}_C = -100\text{j}
\]

Resultant
\[
\vec{R} = \sum \vec{F} = \vec{F}_0 + \vec{F}_A + \vec{F}_B + \vec{F}_C = -125\text{j} - 250\text{j} - 150\text{j} - 100\text{j} = -625\text{j}
\]

Position vector of Point O, \( \vec{r}_O = 0 \)

" of A \( \vec{r}_A = 4\text{i} \)

" of B \( \vec{r}_B = 4\text{i} + 5\text{k} \)

" of C \( \vec{r}_C = 5\text{k} \)

Moment of \( \vec{F}_0 \) about O is zero

" of \( \vec{F}_A \) about O is \( \vec{r}_A \times \vec{F}_A = 4\text{i} \times (-250\text{j}) = -1000\text{i} \times \text{j} = -1000\text{k} \)

Moment of \( \vec{F}_B \) about O is \( \vec{r}_B \times \vec{F}_B \)

\[
\begin{vmatrix}
1 & j & k \\
4 & 0 & 5 \\
0 & -150 & 0
\end{vmatrix} = 750\text{i} - 600\text{k}
\]

Moment of \( \vec{F}_C \) about O is \( \vec{r}_C \times \vec{F}_C = 5\text{k} \times (-100\text{j}) = -500\text{k} \times \text{j} = 500\text{i} \)

Sum of moment of all forces about O
\[
\sum M_O = 0 + (-1000\text{k}) + (750\text{j} - 600\text{k}) + 500\text{i} = 1250\text{i} - 1600\text{k}
\]
Let coordinates of point $D$, the point of application of resultant be $x_D$ and $z_D$. Position vector of $D$ is, $\vec{r}_D = x_Di + z_Dk$.

$$\vec{r}_D = \vec{R} = -625j$$

Moment of resultant about $O$ is $\vec{r}_D \times \vec{R}$.

$$= (x_Di + z_Dk) \times (-625j)$$

$$= -625 x_D (ixj) - 625 z_D (kxj)$$

$$= -625 x_D k - 625 z_D (-j)$$

$$= -625 x_D k + 625 z_D i$$

Equating this moment of resultant about $O$ and sum of all moments of all forces about $O$.

$$\vec{r}_D \times \vec{R} = \Sigma M_o$$

$$625 z_D i - 625 x_D k = 1250 i - 1600 k.$$

$$625 z_D = 1250$$

$$z_D = 2m$$

$$+ 625 x_D = 1600$$

$$x_D = \frac{1600}{625} = 2.56m$$

Determine the centroid of given shape.
<table>
<thead>
<tr>
<th>Sl. No</th>
<th>( A_i ) (cm²)</th>
<th>( x_i ) (cm)</th>
<th>( y_i ) (cm)</th>
<th>( A_i x_i )</th>
<th>( A_i y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((2+8+2) \times 6 = 72)</td>
<td>6</td>
<td>3</td>
<td>432</td>
<td>216</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{\pi \times 1.6}{2} = 25.13)</td>
<td>6</td>
<td>(\frac{4 \times 4 - \frac{16}{\pi}}{3} = 1.97)</td>
<td>150.78</td>
<td>4.2721</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{2} \times 8 \times 6 = 24)</td>
<td>(\frac{3}{3} = 1)</td>
<td>6 - (\frac{6}{3} = 4)</td>
<td>-9</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{1}{2} \times 3 \times 6 = 9)</td>
<td>12 - 1 = 11</td>
<td>6 - 2 = 4</td>
<td>-99</td>
<td>36</td>
</tr>
<tr>
<td>Total</td>
<td>28.87</td>
<td></td>
<td></td>
<td>173.22</td>
<td>101.279</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{173.22}{28.87} = 6 \text{ cm}
\]

\[
\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{101.279}{28.87} = 3.5 \text{ cm}
\]

Find the centroid of the given shape.
<table>
<thead>
<tr>
<th>No.</th>
<th>(A_i) (cm²)</th>
<th>(x_i) (cm)</th>
<th>(y_i) (cm)</th>
<th>(A_i x_i)</th>
<th>(A_i y_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8500</td>
<td>10</td>
<td>12.5</td>
<td>85000</td>
<td>6250</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{\pi}{2} \times 100)</td>
<td>10</td>
<td>25 - (\frac{4 \times 10}{\pi})</td>
<td>1570.8</td>
<td>3259.75</td>
</tr>
<tr>
<td></td>
<td>157.09</td>
<td></td>
<td>20.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>343.8</td>
<td></td>
<td></td>
<td>3430</td>
<td>2992.25</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{10}{343.8} \text{ cm}
\]
\[
\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{8.72}{343.8} \text{ cm}
\]

Find the centre of gravity.
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>$A_i$ (cm²)</th>
<th>$x_i$ (cm)</th>
<th>$y_i$ (cm)</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16×2=32</td>
<td>8</td>
<td>1</td>
<td>256</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>12×2=24</td>
<td>8</td>
<td>8</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>3</td>
<td>8×2=16</td>
<td>8</td>
<td>15</td>
<td>120</td>
<td>240</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>72</strong></td>
<td></td>
<td></td>
<td><strong>576</strong></td>
<td><strong>464</strong></td>
</tr>
</tbody>
</table>

$$
\bar{x} = \frac{576}{72} = 8 \text{ cm}
$$

$$
\bar{y} = \frac{464}{72} = 6.4 \text{ cm}
$$

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>$A_i$ (cm²)</th>
<th>$x_i$ (cm)</th>
<th>$y_i$ (cm)</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10×5=50</td>
<td>$\frac{10}{2} = 5$</td>
<td>25</td>
<td>2500</td>
<td>12500</td>
</tr>
<tr>
<td>2</td>
<td>40×10=400</td>
<td>30</td>
<td>25</td>
<td>12000</td>
<td>10000</td>
</tr>
<tr>
<td>3</td>
<td>10×10=100</td>
<td>55</td>
<td>35</td>
<td>5500</td>
<td>3500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1000</strong></td>
<td></td>
<td></td>
<td><strong>20000</strong></td>
<td><strong>26000</strong></td>
</tr>
</tbody>
</table>

$$
\bar{x} = \frac{20000}{1000} = 20 \text{ cm}
$$

$$
\bar{y} = \frac{26000}{1000} = 26 \text{ cm}
$$
2) A table is shown with the following columns:

<table>
<thead>
<tr>
<th>Gl. No.</th>
<th>$A_i$ (cm$^2$)</th>
<th>$x_i$ cm</th>
<th>$U_i$ (cm)</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14x10 = 140</td>
<td>5</td>
<td>7</td>
<td>700</td>
<td>980</td>
</tr>
<tr>
<td>2</td>
<td>3x5 = 15</td>
<td>7.5</td>
<td>0 + 2.5 = 4.5</td>
<td>112.5</td>
<td>69.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>125</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Further calculations are shown:

$$\bar{x} = \frac{587.5}{125} = 4.7 \text{ cm}$$

$$\bar{y} = \frac{1047.5}{125} = 8.4 \text{ cm}$$

$$\bar{y} = \frac{912.5}{125} = 7.3 \text{ cm}$$
Moment of inertia is a measure of distribution of area w.r.t. an axis.

Consider a plane of area A as shown in figure. Let y be the axis above which we are finding the moment of area.

Consider small area $dA$ in the plane, the distance of the small area from axis y is $x$.

Then, moment of area $dA$ about axis $y$ is given by $dA \cdot x$.

Then, second moment of area of $dA$ w.r.t. axis $y$ is $dA \cdot x^2$.

Second moment of area of plane about axis $y$ is $\int dA \cdot x^2$.

Second moment of area is also called as area moment of inertia.

Moment of inertia of a plane w.r.t. a vertical axis $y$ is given by $I_{yy} = \int dA \cdot x^2$. 
Moment of inertia of a plane about a horizontal axis \( xx \) is given by

\[
I_{xx} = \int dA \cdot y^2
\]

**Theorem of Moment of Inertia**

1) **Perpendicular Axis Theorem**

It states that if \( I_{x-x} \) and \( I_{y-y} \) are the moment of inertia of a plane about two mutually perpendicular axes \( x-x \) \& \( y-y \) passing through the plane. Then moment of inertia of that plane about axis \( z-z \) which is \( \perp \) to the plane and passing through the point of intersection of \( x-x \) \& \( y-y \) is given by

\[
I_{z-z} = I_{x-x} + I_{y-y}
\]

Consider a plane of area \( A \) as shown in figure

\[ x-x \perp y-y \] are two axes as shown.

\( \bar{x} \) is the distance of centroid from axis \( y-y \) and \( \bar{y} \) is the distance below centroid and horizontal axis.
Then we have
\[ I_{x-x} = A \cdot y^2 \]
\[ I_{y-y} = A \cdot \bar{x}^2 \]

Moment of inertia of the plane about axis z-z to the plane and passing through point 0 is given by \( I_{z-z} = A \cdot r^2 \)

we have, \( r^2 = \bar{x}^2 + \bar{y}^2 \)

\[ I_{z-z} = A \left( \bar{x}^2 + \bar{y}^2 \right) \]
\[ = A \cdot \bar{x}^2 + A \cdot \bar{y}^2 \]

\[ I_{z-z} = I_{y-y} + I_{x-x} \]

2) **Parallel Axis Theorem**

If \( I_a \) is the moment of inertia of the plane about centroidal axis passing through the plane, then moment of inertia of that plane about any other plane parallel to centroidal axis is given by \( I_{A-B} = I_a + Ah^2 \) where \( h \) is the distance below centroidal axis and axis A-B.
Derivation of Parallel axis theorem.

Consider a plane of area $A$ as shown in Fig. Centroidal axis and another axis $AB$ at a distance of $h$ metre from the centroidal axis is shown.

Consider a small area $dA$ at a distance of $y$ from the centroidal axis. Then centroidal moment of inertia $I_a$ is given by

$$I_a = \int dA \cdot y^2$$

Moment of inertia of the plane about axis $A-B$ is given by

$$I_{A-B} = \int dA \cdot (h+y)^2$$

$$= \int dA \cdot (h^2 + 2hy + y^2)$$

$$= \int dA \cdot h^2 + 2h \int y dA + \int y^2 dA$$

$$= h^2 \int dA + 2h \int y dA + \int y^2 dA$$

$\int y dA = 0$ because area above the centroidal axis will be equal to area below it. They will get cancelled each other.

$$I_{A-B} = I_a + Ah^2$$
Determine the moment of inertia of given L section about its centroidal axis.

### To find centroid

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Ai</th>
<th>x̄</th>
<th>ȳ</th>
<th>Aix̄</th>
<th>Aiy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>5</td>
<td>3</td>
<td>300</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>14×4=56</td>
<td>2</td>
<td>4.6=13</td>
<td>112</td>
<td>728</td>
</tr>
<tr>
<td>Total</td>
<td>116</td>
<td></td>
<td></td>
<td>412</td>
<td>908</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{412}{116} = 3.5 \text{ cm}
\]

\[
\bar{y} = \frac{908}{116} = 7.82 \text{ cm}
\]

### To find moment of inertia

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>xi-x</th>
<th>yi-y</th>
<th>Ai(xi-x)²</th>
<th>Ai(yi-y)²</th>
<th>(Ix)_{x-y}</th>
<th>(Ix)_{x-y}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>-4.8</td>
<td>135</td>
<td>180</td>
<td>6×10⁻³</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>-1.5</td>
<td>5.2</td>
<td>126</td>
<td>1514.2</td>
<td>6×10⁻³</td>
<td>1514.2</td>
</tr>
</tbody>
</table>
Moment of inertia of L section about horizontal axis
\[ I_{x-x} = \sum I_u(x-x) + \sum A_i (y_i - \bar{y})^2 \]
\[ = 3991.2 \text{ cm}^4 \]
Moment of inertia about vertical axis
\[ I_{x-y} = \sum I_u(y-y) + \sum A_i (x_i - \bar{x})^2 \]
\[ = 835.6 \text{ cm}^4 \]

Moments of Inertia of Some Standard Shapes

1. Rectangle

Moment of inertia of a rectangle about an axis passing through the centroid and which is parallel to the base:

\[ I_{x-x} = \frac{bh^3}{12} \]

Moment of inertia of a rectangle about a vertical axis passing through the centroid:

\[ I_{y-y} = \frac{hb^3}{12} \]
Moment of inertia of a rectangle about a parallel axis passing through its base.

\[ I_{x-x} = \frac{bh^3}{3} \]

Moment of inertia of a rectangle about a vertical axis passing through an edge.

\[ I_{y-y} = \frac{h^3b^3}{3} \]

2. Triangle

Moment of inertia of a triangle about a horizontal axis passing through its centroid.

\[ I_{x-x} = \frac{bh^3}{36} \]

Moment of inertia of a triangle about a horizontal axis passing through its base.
3. **Circle**

Moment of inertia of a circle about a horizontal axis passing through its centre.

\[ I_{x-x} = \frac{\pi D^4}{64} \]

Moment of inertia of a circle about a vertical axis passing through its centroid.

\[ I_{y-y} = \frac{\pi D^4}{64} \]

Determine the moment of inertia of given I about its centroidal axis.
A. To find centroid.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>( A_i ) (mm²)</th>
<th>( x_i ) (mm)</th>
<th>( y_i ) (mm)</th>
<th>( \bar{x}x_i )</th>
<th>( A_iy_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>48</td>
<td>6</td>
<td>2</td>
<td>288</td>
<td>96</td>
</tr>
<tr>
<td>2.</td>
<td>40</td>
<td>6</td>
<td>9</td>
<td>240</td>
<td>360</td>
</tr>
<tr>
<td>3.</td>
<td>48</td>
<td>6</td>
<td>16</td>
<td>288</td>
<td>768</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>136</strong></td>
<td></td>
<td></td>
<td><strong>816</strong></td>
<td><strong>1224</strong></td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{816}{136} = 6 \text{ mm}
\]

\[
\bar{y} = \frac{1224}{136} = 9 \text{ mm}
\]

To find moment of inertia.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>( x_i - \bar{x} )</th>
<th>( y_i - \bar{y} )</th>
<th>( A_i (x_i - \bar{x})^2 )</th>
<th>( A_i (y_i - \bar{y})^2 )</th>
<th>( (I_4)_{x-x} )</th>
<th>( (I_4)_{y-y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0</td>
<td>-7</td>
<td>0</td>
<td>2352</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[= \frac{12 \times 4^3}{12} = 64 ]</td>
<td></td>
<td></td>
<td></td>
<td>[= \frac{4 \times 12}{12} ]</td>
<td>[= 576 ]</td>
</tr>
<tr>
<td>2.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[= \frac{4 \times 10^3}{12} = 533.3 ]</td>
<td></td>
<td></td>
<td></td>
<td>[= \frac{10 \times 4^3}{12} ]</td>
<td>[= 533.3 ]</td>
</tr>
<tr>
<td>3.</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>2352</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[= \frac{4 \times 12^3}{12} = 64 ]</td>
<td></td>
<td></td>
<td></td>
<td>[= \frac{3 \times 4^3}{12} ]</td>
<td>[= 576 ]</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>0</td>
<td>4704</td>
<td>461.3</td>
<td>1805.3</td>
</tr>
</tbody>
</table>
Moment of inertia about horizontal axis
\[ I_{yx} = \sum (I_a)_{yx} + \sum A_i (y_i - \bar{y})^2 \]
\[ = 461.3 + 470.4 \]
\[ = 931.7 \text{ mm}^4 \]

Moment of inertia about vertical axis
\[ I_{xy} = \sum (I_a)_{xy} + \sum A_i (x_i - \bar{x})^2 \]
\[ = 1205.3 + 0.4 \]
\[ = 1205.7 \text{ mm}^4 \]

A. Determine the moment of inertia of given section about its centroidal axis.

To find centroid:

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Ai</th>
<th>xi</th>
<th>yi</th>
<th>Aixi</th>
<th>Aiyi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>5</td>
<td>14</td>
<td>100</td>
<td>260</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>5</td>
<td>6</td>
<td>120</td>
<td>144</td>
</tr>
<tr>
<td>Total</td>
<td>44</td>
<td></td>
<td></td>
<td>220</td>
<td>404</td>
</tr>
</tbody>
</table>
\[
\bar{x} = \frac{920}{44} \times 46.34 = 5 \text{ mm}
\]
\[
\bar{y} = \frac{404}{44} = 9.18 \text{ mm}
\]

To find moment of inertia:

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>(x_i - \bar{x})</th>
<th>(y_i - \bar{y})</th>
<th>(A_i (x_i - \bar{x})^2)</th>
<th>(A_i (y_i - \bar{y})^2)</th>
<th>((I_w)_{x-x})</th>
<th>((I_w)_{y-y})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3.8</td>
<td>0</td>
<td>288.8</td>
<td>(10 \times 2^3) (12)</td>
<td>(6.6)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-3.2</td>
<td>0</td>
<td>245.76</td>
<td>(10 \times 0^3) (12)</td>
<td>(288)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0</td>
<td>534.56</td>
<td></td>
<td>174.6</td>
</tr>
</tbody>
</table>

Moment of inertia about horizontal axis:

\[
I_{x-x} = \sum (I_w)_{x-x} + \sum A_i (y_i - \bar{y})^2
\]

\[
= 294.6 + 534.56 = 829.16 \text{ mm}^4
\]

Moment of inertia about vertical axis:

\[
I_{y-y} = \sum (I_w)_{y-y} + \sum A_i (x_i - \bar{x})^2
\]

\[
= 174.6 + 0 = 174.6 \text{ mm}^4
\]

Calculate the moment of inertia of the section shown in fig:

![Diagram](image)
To find centroid:

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>$A_i$</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>12</td>
<td>4.8</td>
<td>8</td>
<td>57.6</td>
<td>96</td>
</tr>
<tr>
<td>2.</td>
<td>4.8</td>
<td>8</td>
<td></td>
<td>38.4</td>
<td>14.4</td>
</tr>
<tr>
<td>3.</td>
<td>6.01</td>
<td>14.1</td>
<td></td>
<td>84.13</td>
<td>4.23</td>
</tr>
<tr>
<td>Total</td>
<td>74.1</td>
<td></td>
<td></td>
<td>601.32</td>
<td>210.3</td>
</tr>
</tbody>
</table>

$\bar{x} = 9.11$

$\bar{y} = 2.83$

To find moment of inertia:

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>$x_i - \bar{x}$</th>
<th>$y_i - \bar{y}$</th>
<th>$A_i (x_i - \bar{x})^2$</th>
<th>$A_i (y_i - \bar{y})^2$</th>
<th>$I_{A_i x-x}$</th>
<th>$I_{A_i y-y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>-5.5</td>
<td>-0.8</td>
<td>363</td>
<td>7.68</td>
<td>642/36=10.6</td>
<td>6x4^3/12=16</td>
</tr>
<tr>
<td>2.</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.48</td>
<td>1.92</td>
<td>6x8^3/12=256</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>5.1</td>
<td>0.2</td>
<td>366.741</td>
<td>6.564</td>
<td></td>
<td>1038.621</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>730.221</td>
<td>10.164</td>
<td>199.8</td>
<td>298.4</td>
</tr>
</tbody>
</table>

$I_{x-x} = \sum I_{A_i x-x} + \sum A_i (y_i - \bar{y})^2 = 209.964 \text{ cm}^4$

$I_{y-y} = \sum I_{A_i y-y} + \sum A_i (x_i - \bar{x})^2 = 1028.621 \text{ cm}^4$
Determine the moment of inertia for the area shown in Fig.

A. To find centroid

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>$A_i$</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$A_i x_i$</th>
<th>$A_i y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>10</td>
<td>12.5</td>
<td>5000</td>
<td>6250</td>
</tr>
<tr>
<td>2</td>
<td>151.07</td>
<td>10</td>
<td>$25 - \frac{4 \times 10}{3\pi}$</td>
<td>1570</td>
<td>3257.75</td>
</tr>
<tr>
<td>Total</td>
<td>343.07</td>
<td></td>
<td></td>
<td>3480</td>
<td>29982.25</td>
</tr>
</tbody>
</table>

$\bar{x} = 10 \text{ cm}$

$\bar{y} = 8.72 \text{ cm}$

To find moment of inertia

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>$(x_i - \bar{x})$</th>
<th>$(y_i - \bar{y})$</th>
<th>$A_i (x_i - \bar{x})^2$</th>
<th>$A_i (y_i - \bar{y})^2$</th>
<th>$I_x = \frac{1}{12} \sum (x_i - \bar{x})^2$</th>
<th>$I_y = \frac{1}{12} \sum (y_i - \bar{y})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3.8</td>
<td>0</td>
<td>7220.5</td>
<td>$25\pi \frac{23}{12}$</td>
<td>$1666.16$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>12.05</td>
<td>0</td>
<td>22806.95</td>
<td>$29968.5$</td>
<td>$20393.33$</td>
</tr>
<tr>
<td>Total</td>
<td>15.85</td>
<td>-8.25</td>
<td>0</td>
<td>-15586.95</td>
<td>$22114.7$</td>
<td>$12784.7$</td>
</tr>
</tbody>
</table>
\[ I_{x-x} = \sum (I_{ix})_{x-x} + \sum A_i (y_i - \bar{y})^2 \]
\[ = 59995.45 \text{ cm}^4 + \frac{6527.75 \text{ cm}^4}{2} \]
\[ I_{y-y} = \sum (I_{iy})_{y-y} + \sum A_i (x_i - \bar{x}) \]
\[ = 80599.5 \text{ cm}^4 \]
\[ = 12739.7 \text{ cm}^4 \]

Find the moment of inertia about the centroidal axis for the area shown in fig.

\[ \text{To find centroid} \]

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>A_i</th>
<th>x_i</th>
<th>y_i</th>
<th>A_i x_i</th>
<th>A_i y_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>20</td>
<td>25</td>
<td>40000</td>
<td>50000</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
<td>30</td>
<td>37.5</td>
<td>15000</td>
<td>18750</td>
</tr>
<tr>
<td>Total</td>
<td>1500</td>
<td></td>
<td></td>
<td>25000</td>
<td>31250</td>
</tr>
</tbody>
</table>

\[ \bar{x} = 16.6 \]
\[ \bar{y} = 20.8 \]
To find moment of inertia

<table>
<thead>
<tr>
<th>Gl. No.</th>
<th>x_i - \bar{x}</th>
<th>y_i - \bar{y}</th>
<th>A_i (x_i - \bar{x})^2</th>
<th>A_i (y_i - \bar{y})^2</th>
<th>I_{x-x}</th>
<th>I_{y-y}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.4</td>
<td>4.2</td>
<td>23120</td>
<td>35280</td>
<td>40x50^3</td>
<td>50x40^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>48608</td>
<td>148176</td>
<td>41666.6</td>
<td>26666.6</td>
</tr>
<tr>
<td>2</td>
<td>13.4</td>
<td>16.7</td>
<td>89780</td>
<td>139445</td>
<td>20x25^3</td>
<td>28x20^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>70412</td>
<td>26041.6</td>
<td>16666.6</td>
<td>16666.6</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>-11472</td>
<td>-104165</td>
<td>39062.5</td>
<td>250000</td>
</tr>
</tbody>
</table>

\[ I_{x-x} = \sum I_{x-x} + \sum A_i (y_i - \bar{y})^2 = 286460 \text{ cm}^4 \]

\[ I_{y-y} = \sum I_{y-y} + \sum A_i (x_i - \bar{x})^2 = 188340 \text{ cm}^4 \]

**Note:**
Moment of inertia of a semicircle is as shown below.

a) Moment of inertia about a horizontal axis passing through the base of semicircle.

\[ (I_y)_{x-x} = \frac{\pi D^4}{128} \]

b) Moment of inertia about a horizontal axis passing through the centroid.

\[ (I_y)_{x-x} = 0.11 \text{ k}^4 \]
(I_y)_{y,v} = \frac{\pi D^4}{128}

Determine the moment of inertia of the shaded area in the figure.

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Ai</th>
<th>Yi</th>
<th>Aiyi</th>
<th>( \Delta xi )</th>
<th>( \Delta yi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8400</td>
<td>60</td>
<td>35</td>
<td>50,400</td>
<td>29,400</td>
</tr>
<tr>
<td>2</td>
<td>1050</td>
<td>10</td>
<td>70 - \frac{70}{3} = 46.6</td>
<td>10,700</td>
<td>4,893</td>
</tr>
<tr>
<td>3</td>
<td>1050</td>
<td>110</td>
<td>46.6</td>
<td>115,500</td>
<td>48,930</td>
</tr>
<tr>
<td>4</td>
<td>6300</td>
<td></td>
<td></td>
<td>87,800</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2513.3</td>
<td>60</td>
<td>\frac{4 \times 16}{3\pi} = 16.97</td>
<td>150,798</td>
<td>42,474.7</td>
</tr>
<tr>
<td>Total</td>
<td>3786.7</td>
<td></td>
<td></td>
<td>22,702</td>
<td>153,665.8</td>
</tr>
</tbody>
</table>
\[
\bar{x} = 60 \text{ mm} \\
\bar{y} = 40.58 \text{ mm}
\]

To find the moment of inertia

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>(x_i - \bar{x})</th>
<th>(y_i - \bar{y})</th>
<th>(A_i (x_i - \bar{x})^2)</th>
<th>(A_i (y_i - \bar{y})^2)</th>
<th>(I_{x,x})</th>
<th>(I_{y,y})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-5.6</td>
<td>0</td>
<td>2684.94</td>
<td>1071.763</td>
<td>1071.763</td>
</tr>
<tr>
<td>2</td>
<td>-50</td>
<td>6</td>
<td>2625000</td>
<td>37800</td>
<td>3430000</td>
<td>107.30(^3)</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>6</td>
<td>2625000</td>
<td>37800</td>
<td>30x70(^3)</td>
<td>52x80(^4)</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-23.7</td>
<td>0</td>
<td>141169.5(^5)</td>
<td>0.11x4(^4)</td>
<td>281600</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>-3250000</td>
<td>-122871.5(^5)</td>
<td>2576734(^4)</td>
<td>8969670.4</td>
</tr>
</tbody>
</table>

\[
I_{x,y} = \sum (A_i) x_i y_i + \sum A_i (y_i - \bar{y})^2 \]
\[
= 1352861.9 \text{ mm}^4
\]

\[
I_{y,y} = \sum (A_i) y_i y_i + \sum A_i (x_i - \bar{x})^2 \]
\[
= 3719690.4 \text{ mm}^4
\]
Polar Moment of Inertia

It is the moment of inertia of a plane about an axis at to the plane and passing through the centroid of the plane. It is given by

\[ I = \int dA \cdot r^2 \]

where \( r \) is the distance from the axis and area \( dA \).

By perpendicular axis theorem we have,

\[ I_{zz} = I_{xx} + I_{yy} \]

So polar moment of inertia is equal to the sum of moment of inertia about horizontal axis \( + \) vertical axis.

Polar Moment of Inertia of a Rectangle

We have:

\[ I_{xx} = \frac{bh^3}{12} \]
\[ I_{yy} = \frac{bh^3}{12} \]

Polar moment of inertia,

\[ I = I_{xx} + I_{yy} = \frac{bh^3}{12} + \frac{bh^3}{12} = \frac{bh^3}{6} + \frac{bh^3}{6} = \frac{bh^3}{3} \]
Polar Moment of Inertia of a Circular Section

\[ I_{x-x} = \frac{\pi D^4}{64}, \quad I_{y-y} \]

\[ I = \frac{\pi D^4}{32}. \]

Product of Inertia

Product of inertia is a measure of distribution of area w.r.t a set of mutually perpendicular axes. It is given by

\[ I_{x-y} = \int xy \, dA \]

Product of inertia will be zero when the area is symmetrical to any of the axes or both.

Mass Moment of Inertia (\( I_m \))

It represents the distribution of mass of a body w.r.t an axis. Its symbol is “\( I_m \)”. It is given by

\[ I_m = \int dm \cdot r^2 \]
In the figure dm is a small element on the body and \( r \) is its distance from axis \( xx \).

**Radius of Gyration**

Radius of gyration is represented by symbol \( k \). It is given by \( I_m = mk^2 \) where \( I_m \) is mass moment of inertia of the body and \( m \) is the mass of the body. It is a single parameter for representing the distance of a body from an axis.

Radius of gyration can be defined for a surface as \( I = AK^2 \) where \( I \) is area moment of inertia of the plane and \( A \) is total area of the plane.

\[
k = \sqrt{\frac{I_m}{M}} \quad \text{(mass moment of inertia)}
\]

\[
k = \sqrt{\frac{I}{A}} \quad \text{(for plane)}
\]

\Rightarrow \text{Mass moment of inertia for some standard shapes}

1) Mass moment of inertia of a rectangular block, w.r.t a horizontal axis passing through its centre of gravity.

\[
(I_m)_{x-x} = \frac{m h^2}{12}.
\]

2) Mass moment of inertia of a rectangular block, w.r.t a horizontal axis passing through its space.
3) Mass moment of inertia of a rectangular block about a vertical axis passing through its opposite edge.

\[ (I_m)_{x-y} = \frac{mh^2}{3} \]

4) Mass moment of inertia of a rectangular block about a vertical axis passing through its centroid.

\[ (I_m)_{y-y} = \frac{mb^2}{3} \]

5) Mass moment of inertia of a rectangular block about an axis perpendicular to the block and passing through the centre of gravity.

\[ (I_m)_{z-z} = I_x - x + I_y - y = \frac{m}{12} (h^2 + b^2) \]
THEOREM OF PAPPUS + GULDINS

Theorem 1

The area of surface generated by revolving a plane curve about a non-intersecting axis in the plane of the curve is equal to the product of length of curve and the distance travelled by the centroid 'C' of the curve during revolution.

\[ \text{Surface of revolution is generated by rotating a plane curve about a fixed axis known as the axis of revolution.} \]

Theorem 2

The volume of solid generated by revolving a plane area about a non-intersecting axis in its plane is equal to the product of area and length of the path travelled by the centroid 'C' of the area during the rotation about the axis.

\[ \text{The body of revolution is generated by rotating a plane area about a fixed axis known as the axis of revolution.} \]

Determine the mass moment of inertia of the composite solid shown in Fig. - through an axis passing through the centre of rectangular block and

given density of rectangular block is 6000 kg/m³.

density of cylinder 8000 kg
mass of rectangular block = density \times volume
\[
= 6000 \times 120 \times 50 \times 60 = 6000 \times 0.08 \times 0.06 \times 0.06 \\
= 2.16 \text{ kg}
\]

mass of cylinder = \(8000 \times \pi \times (0.02)^2 \times 0.08\)
\[
= 0.8042 \text{ kg} \approx 0.8 \text{ kg}
\]

moment of inertia of the rectangular block w.r.t an axis \( z \) to it and passing through the centre of gravity is given by
\[
(I_m)_{Z-Z} = \frac{m}{12} (h^2 + b^2)
\]
\[
= \frac{2.16}{12} (120^2 + 60^2) \\
= 0.003042 \text{ kg.m}^2
\]

mass moment of inertia of a cylinder w.r.t an axis passing through its centre is given by
\[
I_m = \frac{MR^2}{2}
\]
\[
= \frac{0.8 \times 0.02^2}{2} \\
= 0.00016 \text{ kg.m}^2
\]

To transfer this moment of inertia to the axis passing through the centre of gravity of composite block, we have parallel axis theorem for mass moment of inertia as,
\[
(I_m) = (I_c) + m \cdot h^2
\]
where two axes,
\[
(I_m)_{Z-Z} = (I_c) \text{ cylinder } + Mh^2
\]
\[
= 0.00016 + 0.8 \times (0.060 + 0.09)^2 \\
= 0.00016 + 0.8 \times 0.04 \text{ kg.m}^2
\]
\[
= 0.00144 \text{ kg.m}^2
\]

mass moment of inertia of composite solid w.r.t given axis
\[
\Phi (I_m)_{Z-Z} \text{ of rectangular } \Phi (I_m)_{Z-Z} \text{ of cylinder}
\]
\[
= 0.004482 \text{ kg.m}^2
\]